the entire year. Next is the west-central district, with a decrease of 2.97 inches, which coincides with the impressions of farmers who have farmed in that district for a lifetime and who furnished part of the inspiration for this study. The least decrease in summer rainfall is in the north-central district, 1.06 inches.

The values of the ends of the trend lines of the nine districts were entered on maps to show just how the summer precipitation of 53 years ago would compare with

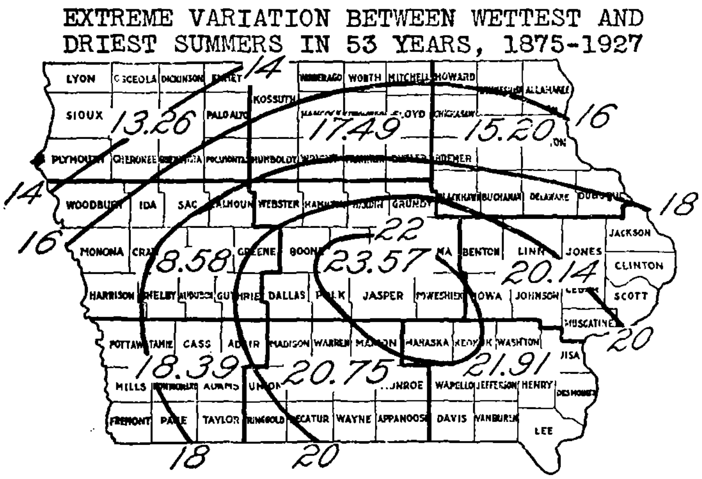

FIGURE 11.—The central district shows the most extreme variation in summer rainfall while the northwest district is most constant

that of 1927. These are shown by maps. (Figs. 7 and 8.) Note the center of heavy rainfall 53 years ago in the southwest and the area of relatively heavy rainfall extending northeast beyond the center of the State. Note also that at the end of the period the differences have become leveled down so that there is now a range of only 0.64 inch between the driest northwest district and the wettest south-central district, while 53 years ago the range was 2.34 inches between the driest northwest district and the wettest southwest district.

Corn, if affected at all by this decrease in summer rainfall, has been benefited, or possibly improved farming has more than overcome any adverse effects, for the trend in corn yield per acre has been upward in all districts for 40 years, though the improvement in corn has been least in the southwest and south-central districts, where it amounts to only a gain of 0.18 bushel per acre per year. The greatest concentration of acres of corn per unit area is in the western districts, so there was evidently much superfluous rainfall in the earlier years.

The record wet summer was 1902 in all districts, except the west-central, where the summer of 1875 was the wettest, and the northwest, where the summer of 1900 was the wettest; and the wettest district was the central in 1902 with 26.21 inches. (Fig. 9.) The record dry summer was 1886 in all but the southwest district, where the driest was 1911, with 4.40 inches, and the southeast, where the driest was 1894, with 3.97 inches; and the driest district was the central in 1886, with 2.64 inches. (Fig. 10.) In extreme variation of summer rainfall the central district leads with a range of 26.21 inches, and the northwest district is least with 13.26 inches. (Fig. 11.) This small variation in rainfall, combined with a drouth-resistant soil, explains the dependability of the northwest district in corn production. At the end of the trend period in 1927 the northwest district has 38 per cent of its annual rainfall in the summer, leading in this feature over all the other districts, while the east-central district has the least, 33 per cent. At the beginning of the period the west-central district led with 44 per cent, and the east-central and southeast were least with 37 per cent.

SUMMARY

Iowa is becoming steadily drier, but up to this time the tendency has not proceeded far enough to threaten its principal crop, corn; in fact, conditions for corn seem to be improving. There is, no doubt, a limit, but probably the trend will change before the danger line for corn is reached.

ALIGNMENT DIAGRAM FOR "R" OF THE ENERGY-EVAPORATION EQUATION

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The evaporation from an open-water surface is determined by the relation (see Cummings and Richardson, 1927).

$$E=\frac{I-B-S-C}{L(1+R)} \tag{1}$$

where (E) is the evaporation in centimeters of depth, during any arbitrary time, (I) is the energy which the sun and sky deliver to 1 square centimeter of the water surface during the same interval of time; (S) is the energy which accumulates during this time in a column of water having an area of 1 square centimeter and a depth equal to the average depth of the water body, while (C) takes care of certain corrections which are usually small. (L) is the latent heat of water and (R) is the ratio of sensible heat swept away by the wind to latent heat carried off by the vapor. Bowen (1926) shows that (R) can be calculated by means of the equation

$$R=.46\frac{T_w-T_a}{P_w-P_a}\frac{B}{760} \tag{2}$$

where (T_w) is the temperature of the water surface, (T_a) that of the dry bulb, (P_w) the pressure of saturated water vapor at temperature (T_w), while (P_a) is the absolute humidity in millimeters of mercury, and (B) is the barometric pressure.

In order to facilitate numerical applications an alignment diagram was constructed by the following method:

Assuming $B=760$ the equation may be written in the determinant form

$$\begin{vmatrix} 1 & R & 0 \\ P_a & .46T_a & 1 \\ P_w & .46T_w & 1 \end{vmatrix}=0 \tag{3}$$

Making use of the fact that the elements of any row or column can be added to the element of any other row or column without changing the value of the determinant, we add the elements of the first column to those of the last, obtaining

$$\begin{vmatrix} 1 & R & 1 \\ P_a & .46T_a & 1+P_a \\ P_w & .46T_w & 1+P_w \end{vmatrix}=0 \tag{4}$$

Making use of the further fact that the effect of multiplying or dividing all the elements of any column or row by the same number is to multiply or divide the determinant by that number, we divide the second row by $(1+P_a)$, and the third by $(1+P_w)$, thus obtaining

$$\begin{vmatrix} 1 & R & 1 \\ \frac{P_a}{1+P_a} & \frac{.46T_a}{1+P_a} & 1 \\ \frac{P_w}{1+P_w} & \frac{.46T_w}{1+P_w} & 1 \end{vmatrix} = 0 \tag{5}$$

It is a property of determinants that if any two rows are interchanged the sign of the determinant is changed. We can therefore interchange the first and third rows, and the the first and second rows of the resulting determinant, obtaining finally

$$\begin{vmatrix} \frac{P_a}{1+P_a} & \frac{.46T_a}{1+P_a} & 1 \\ \frac{P_w}{1+P_w} & \frac{.46T_w}{1+P_w} & 1 \\ 1 & R & 1 \end{vmatrix} = 0 \tag{6}$$

By adopting the following three pairs of equations

$$\begin{aligned} x_1 &= \frac{P_a}{1+P_a} & y_1 &= \frac{T_a}{1+P_a} \\ x_2 &= \frac{P_w}{1+P_w} & y_2 &= \frac{T_w}{1+P_w} \\ x_3 &= 1 & y_3 &= R \end{aligned} \tag{7}$$

we can write equation (6) in the form

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \tag{8}$$

If the three points (x_1y_1), (x_2y_2), and (x_3y_3) lie on a straight line, their coordinates must satisfy the equation

$$\frac{y_3-y_1}{x_3-x_1} = \frac{y_2-y_1}{x_2-x_1} \tag{9}$$

But equations (8) and (9) are really only two forms of the equation

$$x_1y_2 + x_2y_3 + x_3y_1 - x_3y_2 - x_2y_1 - x_1y_3 = 0$$

Consequently any three points whose coordinates satisfy equation (8) must lie on a straight line.

If, therefore, three loci are platted, one for each pair of equations (7), and if a straight line is laid across these loci, it will intersect them in three points which, taken as a set, satisfy equation (6). But since equation (6) is equivalent to the original equation (2), the three points of intersection must also satisfy equation (2).

If we adopt an arbitrary wet-bulb temperature, (P_a) becomes definite whenever the dry-bulb temperature is known, and (P_w) is, of course, determined by the water-surface temperature alone. We might therefore construct the chart after computing Table 1 by proceeding as follows:

1. Adopt an arbitrary wet-bulb temperature and plot the first pair of equations (7) regarding (T_a) as a variable parameter.
2. Choose a different wet-bulb temperature and repeat step 1. Repeat this for the entire range of wet-bulb temperatures.
3. Plot the locus of the second pair of equations (7) regarding (T_w) as the variable parameter. Call the resulting line the water-temperature locus. The necessary computations are shown in Table 1.
4. Plot the last locus from the equations $(x=1)$, $(y=R)$. Call this the (R) locus.

Actually it is unnecessary to plot the first pair of equations point by point because if the water temperature is equal to the wet bulb, then $R=-1$ for all dry-bulb temperatures. Therefore the lines of constant wet-bulb temperature must be straight and each must pass through the point on the water-temperature locus corresponding to the wet bulb to which the line belongs and also through the point on the (R) locus corresponding to $(R=-1)$.

The various points on the lines of constant wet-bulb temperature can easily be found by remembering that

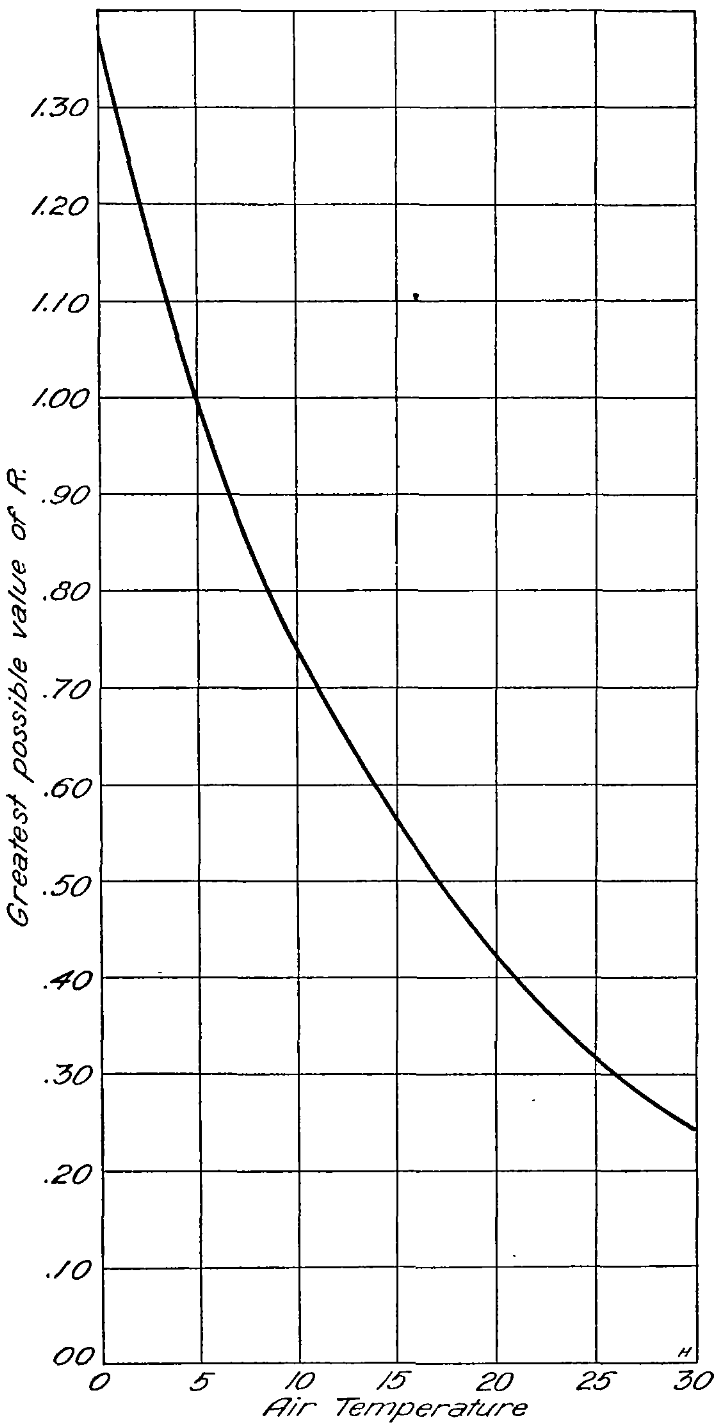

FIGURE 1.—Alignment diagram for the Bowen formula

when the air temperature and water-surface temperature are equal, (R) must be zero. Hence if straight lines are drawn through $(R=0)$ and through each degree point on the water-temperature locus they will intersect the wet-bulb lines at the appropriate dry-bulb temperatures.

The entire net can be plotted in this manner. All the loci are shown in Figure 1.

TABLE 1.—*Values of functions used in constructing the alignment diagram*

T_w=water temperature	P_w=vap. press.	$x=\frac{P_w}{1+P_w}$	$\frac{T_w}{1+P_w}$	$y=\frac{.46T_w}{1+P_w}$
0	4.58	0.8207	0	0
1	4.92	.8310	.1689	.0777
2	5.29	.8410	.3179	.1462
3	5.68	.8502	.4491	.2065
4	6.10	.8591	.5633	.2591
5	6.54	.8673	.6631	.3050
6	7.01	.8751	.7490	.3445
7	7.51	.8824	.8225	.3784
8	8.04	.8893	.8849	.4070
9	8.61	.8959	.9365	.4308
10	9.21	.9020	.9794	.4505
11	9.85	.9078	1.0138	.4663
12	10.52	.9131	1.0416	.4791
13	11.23	.9182	1.0629	.4889
14	11.99	.9230	1.0778	.4958
15	12.79	.9274	1.0877	.5003
16	13.64	.9316	1.0928	.5027
17	14.54	.9356	1.0939	.5032
18	15.49	.9393	1.0915	.5021
19	16.49	.9428	1.0863	.4997
20	17.55	.9460	1.0781	.4959
21	18.66	.9491	1.0681	.4913
22	19.84	.9520	1.0556	.4856
23	21.09	.9547	1.0411	.4789
24	22.40	.9572	1.0256	.4718
25	23.78	.9596	1.0088	.4640
26	25.23	.9618	.9912	.4560
27	26.77	.9639	.9722	.4472
28	28.38	.9659	.9530	.4384
29	30.08	.9678	.9330	.4292
30	31.86	.9695	.9129	.4199
31	33.74	.9712	.8923	.4104
32	35.70	.9727	.8719	.4011
33	37.78	.9742	.8509	.3914
34	39.95	.9755	.8302	.3819
35	42.23	.9768	.8096	.3724
36	44.62	.9780	.7891	.3630
37	47.13	.9792	.7687	.3536

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CERTAIN LIMITATIONS ON THE POSSIBLE VALUES OF THE RATIO OF HEAT LOSSES BY CONVECTION AND BY EVAPORATION AT A WATER SURFACE

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An alignment diagram has recently been prepared for the rapid computation of the ratio of the two quantities of heat leaving a water surface (see preceding paper). In addition to facilitating routine computations the diagram is useful as a rapid means of setting limits on the value that (R) can assume under certain specified conditions which alone are not sufficient for its exact evaluation. The explanation of this use of it will be postponed, however, until certain conclusions based on the general principles of thermodynamics shall have been drawn.

The atmosphere obtains most of its heat from the surface of the earth, and must therefore for the earth as a whole be colder than the earth's surface. Since a large part of the earth is covered with water, the water must in general be warmer than the air at the surface of contact. Although there may be isolated cases in which the air is warmer, they must be regarded as the exception rather than the rule. This deduction agrees with observation. It is, of course, understood that daily averages are referred to; at certain times of day the air is warmer almost anywhere.

Since negative evaporations from large bodies of water are rare it follows that from the meteorological standpoint positive values of (R) are more interesting than negative. Negative values of (R) combined with positive evaporations tend naturally to eliminate themselves because when (R) is negative the water is almost sure to be warming rapidly and (R) is thus in the act of becoming positive, unless very cold water is being supplied rapidly. The discussion will be limited therefore to positive values of ($T_w - T_a$).

Under these conditions evaporation rather than condensation is taking place. For any given air temperature and water temperature, then, (R) will be a maximum when wet and dry bulb temperatures are equal, because this condition makes (P_a) a maximum and therefore makes the denominator of the fraction a minimum, making the fraction as a whole a maximum. It follows that for the purpose of estimating (R_m), the maximum value of (R), (P_a) may be regarded as the pressure of saturated vapor at the temperature of the dry bulb.

For a given air temperature when wet and dry bulb are equal, (R) must increase as (T_w) decreases. This is evident from the fact that for saturated vapor $\frac{\Delta p}{\Delta t}$ decreases as the temperature (t) decreases and consequently reaches a minimum at the limit $\frac{dp}{dt}$. For any given air temperature, therefore, we may easily determine the greatest possible value of (R) by dividing .46 by $\frac{dp}{dt}$.

This derivative or the corresponding (R) may be computed in various ways:

(1) Directly from a table of saturated vapor pressures. This can be done most accurately by numerical differentiation, as described by von Sanden.

(2) From the well-known Clapeyron equation

$$L = t(v_1 - v_2)\frac{dp}{dt}$$